

Logarithmic functions are inverses of exponential functions.

For $b > 0$, these two statements are equivalent

$$(1) y = \log_b x \text{ if and only if } (2) x = b^y$$

In the previous lecture you learned that $\frac{d(b^x)}{dx} = (\ln b)b^x$.

Applying this formula to (2) we have $\frac{dx}{dy} = (\ln b)b^y$

To find the derivative of the function in (1), take reciprocals on both sides:

$$\frac{dy}{dx} = \frac{1}{(\ln b)b^y} = \frac{1}{\ln b} \cdot \frac{1}{x}$$

Note that when $b = e$, the formula becomes $\frac{d(\log_e x)}{dx} = \frac{d(\ln x)}{dx} = \frac{1}{x}$, since $\ln e = 1$.

Compare these with formulas (8-11) and (8-12) on page 273. Notice that the first formula in the book involves $\log_b e$ rather than $\ln b$. Since the calculator gives values of logarithmic functions only for base 10 and base e , using $\ln b$ is preferable.

Graph the function $y = \ln x$. Note the domain and range of this function. Can you see from the graph why the derivative appears to be $\frac{1}{x}$?

Example 1 page 273: $y = \log_4 x$. The base b in this case is 10.

$$\text{Let } u = 4x. \text{ Then } \frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = \frac{1}{\ln 10} \frac{1}{4x} \cdot 4 = \frac{1}{x \ln 10}.$$

Compare this with the book's solution: $\frac{dy}{dx} = \frac{1}{x} \log_e e$. Note that $\log_e e \approx .4343$ and $\ln 10 \approx 2.03$.

Examples 2 and 3 show applications of the derivative of the natural logarithm function.

Example 4 on page 274 shows how the properties of logarithms may be used to simplify a function. Note these properties in the upper left corner of the page:

$$\begin{aligned} \log_b xy &= \log_b x + \log_b y \\ \log_b \left(\frac{x}{y} \right) &= \log_b x - \log_b y \\ \log_b x^n &= n \log_b x \end{aligned}$$

You should recall these from Pre-calculus. Check for these values: $b = 2$, $x = 32$, $y = 8$, $n = 2$, using your familiarity with powers of 2.

In Section 8-4 there are two examples you should study.

Example 1 on page 280 shows an application to curve sketching.

Example 5 on page 281 shows an application involving velocity and acceleration.

Exercises:

page 275: 5, 7, 9, 11, 15, 21

page 282: 1, 5, 21, 27, 31